International Council for the Exploration of the Sea

CM 1977/C:43
Hydrography Committee

SPATIAL VARIABILITY OF RESIDUAL CURRENTS IN AN AREA OF THE SOUTHERN NORTH SEA

H.W. Riepma
K.N.M.I., De Bilt
Netherlands.

ABSTRACT

During the period 21 September until 29 October 1976 four currentmeterrigs were laid out by the KNMI in the Southern Bight of the North Sea on positions shown in figure 1. This was done as a first step for a programme to investigate spatial variability of residual currents on the Dutch part of the continental shelf. In this contribution special attention is given to the use of stream functions to investigate properties of residual currents in an area of 10 x 10 miles for which an objective method to obtain streamline patterns is necessary. Interesting results were obtained comparing measured residual fluxes and residual fluxes calculated by a numerical model.

INTRODUCTION

Residual currents as observed in the open North Sea with some exceptions generally show a reasonable regional coherence, as can be shown from JONSDAP-73 and -76 results. However, looking more in detail, differences do occur that are not without importance. Apart from the question about the background of these differences there is a problem when results from numerical current models have to be compared quantitatively with actual current data. Such model results are valid for larger areas of the size of the grid used in the model which amounts up to some 20 x 20 miles.

An investigation into the regional variation of residual currents and on the possible ways to obtain estimates of the actual regional mean of residual currents therefore are important. A first attempt of this nature is described here, where it is hoped that further study of the results may lead to a number of similar excercises.

MEASUREMENTS.

As mentioned before four currentmeterrigs, each having two currentmeters, were situated on positions shown in table 1 and figure 1 in the period 21 September until 29 October 1976. On each rig one meter was suspended 5 m above bottom and one meter on a depth of about 16 to 18 m below sealevel. Rigs 1,2 and 4 (see figure 1) all had Plessey currentmeters, rig 3 combined a near bottom Plessey currentmeter and a near surface NBA currentmeter. Details are given in table 1.

An estimate of the maximum error for individual mean values of the residual currents (average currents over 24h 50 m) is 20% or 1 cm/s whichever is the greatest. Part of the sources of these errors is stochastic, part is systematic for a certain instrument, but stochastic between different instruments. The area considered varies in depth between 32 and 41 m., with sand waves of some 6 m. height.

A TEST ON LINEAR INTERPOLATION OF FLUXES.

The numerical models presently in existence for the residual currents of the southern North Sea are nearly all "vertically averaged" models, giving vertically averaged residuals or essentially, waterfluxes. For comparison observed currents have to be converted to fluxes. If two observations are made along the vertical, the flux is estimated using the profile shown in figure 2. To this flux a flux originating from averaging instantaneous tidal fluxes has to be added; the necessary data on the vertical tides (mainly M_2) have been taken from (1).

A first check on the spatial variability was made by estimating the degree of divergence in the current field assuming linear variation of the fluxes between pairs of observational points. Because fluxes in this divergence calculation are based on the results of two currentmeters on every rig the period for which results could be obtained was limited to 7 days because of technical failure of some currentmeters afterwards. According to this check, using averages of the fluxes present on the sides of triangles, it appeared that sealevel should have risen of the order of 5% per day of the total waterdepth, which is an absurd value.

Now either errors in the estimate of the fluxes or failure of the linear flux interpolation between two stations must be considered as the cause of this apparent convergence. With a 20% maximum error in the residual current, a maximum error in the diurnal sealevel increase/decrease may occur of 19% of the total waterdepth for the length scales involved. In the long run, however, a much smaller mean error can be expected.

APPLICATION OF STREAM FUNCTIONS

Because generally, and certainly during the experiment, mean sealevel shows minor non-tidal variations (less than 1% of the water depth), use was made of stream functions to analyze the residual current data.
According to this principle we put:

$$F_x = -\frac{\partial\psi}{\partial y} \quad \text{and} \quad F_y = \frac{\partial\psi}{\partial x} \qquad (1)$$

in which F_x= east component of the flux parallel to the x-axis and F_y= north component of the flux parallel to the y-axis, thus having $\frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y=0$.
It was assumed that the streamfunction ψ on a particular day could be approximated as a second order Taylor expansion:

$$\psi = \psi_o+b_1x+b_2\, y+c_{11}x^2+c_{12}\, xy+c_{22}\, y^2 + --- \qquad (2),$$

Fitting such a quadratic stream function is equivalent with fitting a linear function to all 4 observed residual fluxes but with the flux divergence forced to zero.
In equation (2) the unknowns b_1, b_2, c_{11}, c_{12} and c_{22} must be chosen so as to get an optimal fit with measurements. Use has been made of a least square method fitting the fluxes as defined by the stream function (2) with the observed residual fluxes. With four measured residual fluxes it is possible to derive eight equations for the five unknowns b_1, b_2, c_{11}, c_{12} and c_{22}.

The period over which all data can be used extends only over seven days. In order to investigate conditions over a longer period a correlation technique was used, relating the flux at a certain rig with the near surface residual current, according to the following relation

$$\vec{F} = \vec{A} + \vec{\vec{B}}\,\vec{U}_{surface} \qquad (3)$$

in which $\vec{F}$ = flux, $\vec{A} = \begin{pmatrix} a_{north} \\ a_{east} \end{pmatrix}$ = constant,

$\vec{\vec{B}} = \begin{pmatrix} b^o_{north} & b^o_{east} \\ b'_{north} & b'_{east} \end{pmatrix}$ = constant and $\vec{U}_{surface}$ = near surface residual current. $\vec{A}$ and $\vec{\vec{B}}$ were estimated using the days on which both current-meters operated on a rig. In those cases fluxes could be calculated using the profile of figure 2 which then could be used to fit relation (3). Typical results are shown in figure 3 and 4 in which fluxes devided by the total waterdepth are given.

Following this procedure a total of 16 days of useful data could be obtained during which on every rig fluxes were known to which daily stream-functions (2) could be adjusted. Examples of daily streamline patterns are shown in the figures 5 until 9 in which also the flux vectors calculated according to (3) are given. Clearly, on some days, eddy like structures show up with a clock wise fluid rotation (see also below).

On the average the difference between the flux components which can be calculated using relation (1), after having adjusted ψ, and the measured residual flux components equals about 0.2 m^2/s, giving an accuracy of about 0.5 cm/s in terms of vertically averaged residual current components. This accuracy is of the same order compared to the assumed absolute error in measured residual current components, thereby showing that (2) can give a useful fit to current data in a not to large area.

Of course a much larger regional variability on smaller scales might be present, necessitating higher order stream functions. More detailed observations should be used to investigate this possibility. However, for the time being we assume that smaller scale variability is small compared to the variability observed.

THE REGIONAL AVERAGE OF THE RESIDUAL FLUX

By definition it is stated that the regional average of the residual flux $\langle \vec{F} \rangle$ is given by

$$\langle \vec{F} \rangle = \frac{1}{S} \iint dS \, (F_x, \, F_y)$$

in which (F_x, F_y) follow from relations (2) and (3), S = area of triangle (1,2,4).

Straight forward mathematics shows that

$$\langle \vec{F} \rangle = (-b_2, \, b_1)$$

if the origin of coördinates is chosen in the centre of gravity of triangle (1,2,4).

It is interesting to investigate how this regional average agrees with modelresults, compared with fluxes from individual currentmeter stations. A comparison was made with the outcome of the KNMI stormsurge model. This model, described by Timmerman (2), also gives water-movements dependent on meteorological forces (windstress, atmospheric pressure). The day-to-day results for a grid point in the same region are compared with the regional mean flux as described above and with the fluxes from the four rigs. The correlation coëfficiënts between these series of diurnal data (model calculated and observational) are given in table 2. It can be seen from table 2 that the correlation coëfficiënts for rig 1 and 3 are worse compared with the regional average. Correlation coëfficiënts for rig 2 and 4 are roughly the same compared with the regional average. This supports the reservation expressed in the introduction concerning the use of single station data for verification of residual current models.

An example of a scatterdiagram is shown in figure 11 in which estimated measured residual fluxes are plotted against model calculated residual fluxes.

EDDIES IN THE RESIDUAL CURRENT FIELD

From day to day it can be seen from the examples given in figure 5 until 9 that eddies show up in the current field. A preliminary investigation of these eddies was made using stream functions adjusted to near surface residual currents in order to evaluate the current data to vorticity estimates. As the near surface residual currents are only free of divergence under certain conditions the results presented below are only indicative.

From (1) and (2) it is possible to equate relative vorticity ζ to

$$\zeta = \Delta \psi$$

in which Δ = Laplace operator, if F_x and F_y in (1) are replaced by U and V, the residual near surface velocity components. As a function of time ζ is shown in figure 10. Around a mean value of about $-2.5 \times 10^{-6}\ s^{-1}$ a marked variation in ζ takes place. Two extremes show up $2\frac{1}{2}$ to 3 days after spring and neap tide. It can be shown that the maximum error in ζ is about $1.4 \times 10^{-6}\ s^{-1}$, again using a 20% error in residual currents, for the length scales involved. The real error probably will be smaller because of statistically averaging of individual errors over a greater number. A reduction by 3 looks reasonable.

The source of this negative vorticity is not clear. Residues of tidal components in the residual currents after filtering out the M_2-component are too small to account for the observed vorticity. Considering however the time dependence of ζ in figure 10 someother tidal influence is presumed (interaction between tides and bottom topography?).

ACKNOWLEDGEMENT

The author wishes to express his thanks to dr.S. Kruizinga of KNMI for making it possible to adjust stream functions in a way not conflicting with statistical habits of thoughts.

LITERATURE:

1. Sager G. and Sammler R. — Allas der Tidewasserstände Institut für Meereskunde der Deutsche Akademie des Wissenschaften (1964).

2. Timmerman H. — On the importance of Atmospheric Pressure gradients for the generation of External Surges in the North Sea. D.H.Z. 28 1975 (62).

Table 1.

STATION DETAILS

	position	waterdepth (m)	meterdepth (m)	metertype	duration (days)
rig 1	52°8'N 3°16'E	32	18 27	Plessey "	17 7
rig 2	52°12'N 3°1'E	35	16 30	" "	37 16
rig 3	52°15'N 3°11'E	34	16 29	N.B.A. Plessey	31 8
rig 4	52°22'N 3°7'E	41	17 36	" "	16 10

Table 2.

Correlation coëfficients between model calculated residual water fluxes and residual waterfluxes estimated from measurements.

Correlation coëfficiënts (%)

	east components of fluxes	north components of fluxes
rig 1	41	40
rig 2	68	95
rig 3	47	84
rig 4	69	94
regional averaged components derived from linear part of stream functions	64	92

95% confidence level=50%

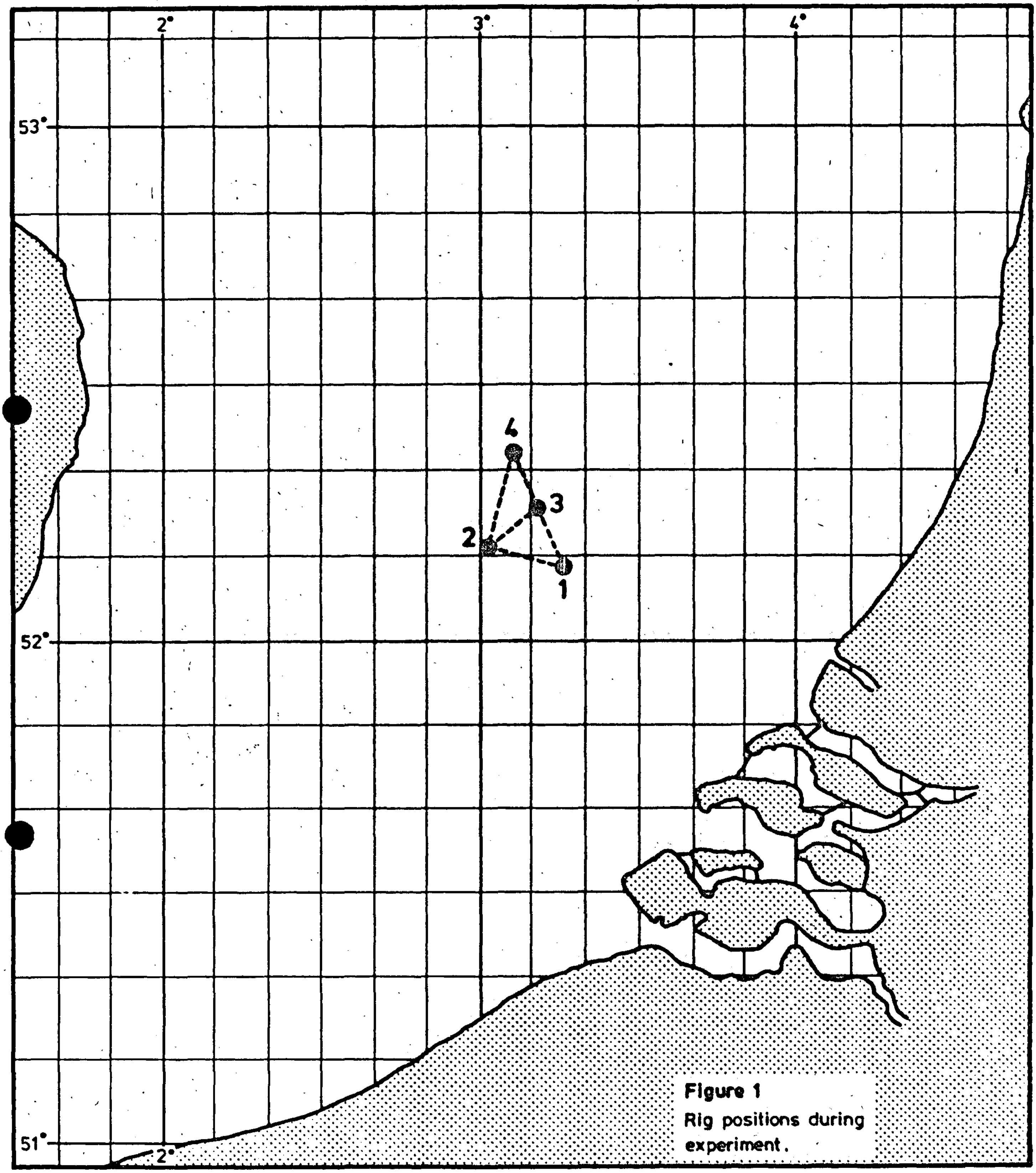

Figure 1
Rig positions during experiment.

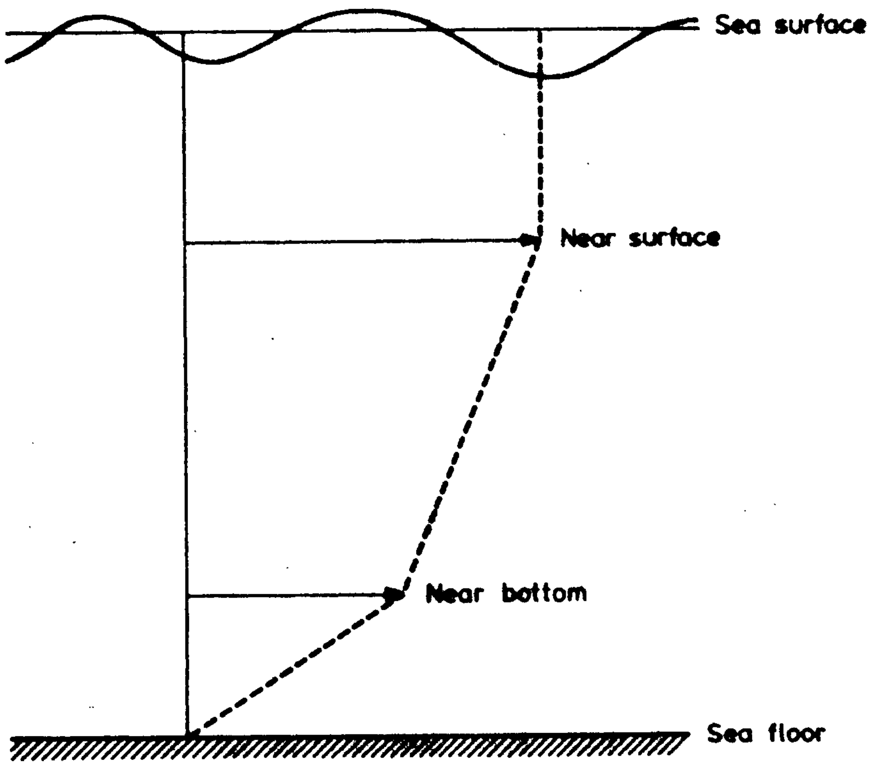

Figure 2

Estimate of residual current as a function of depth.

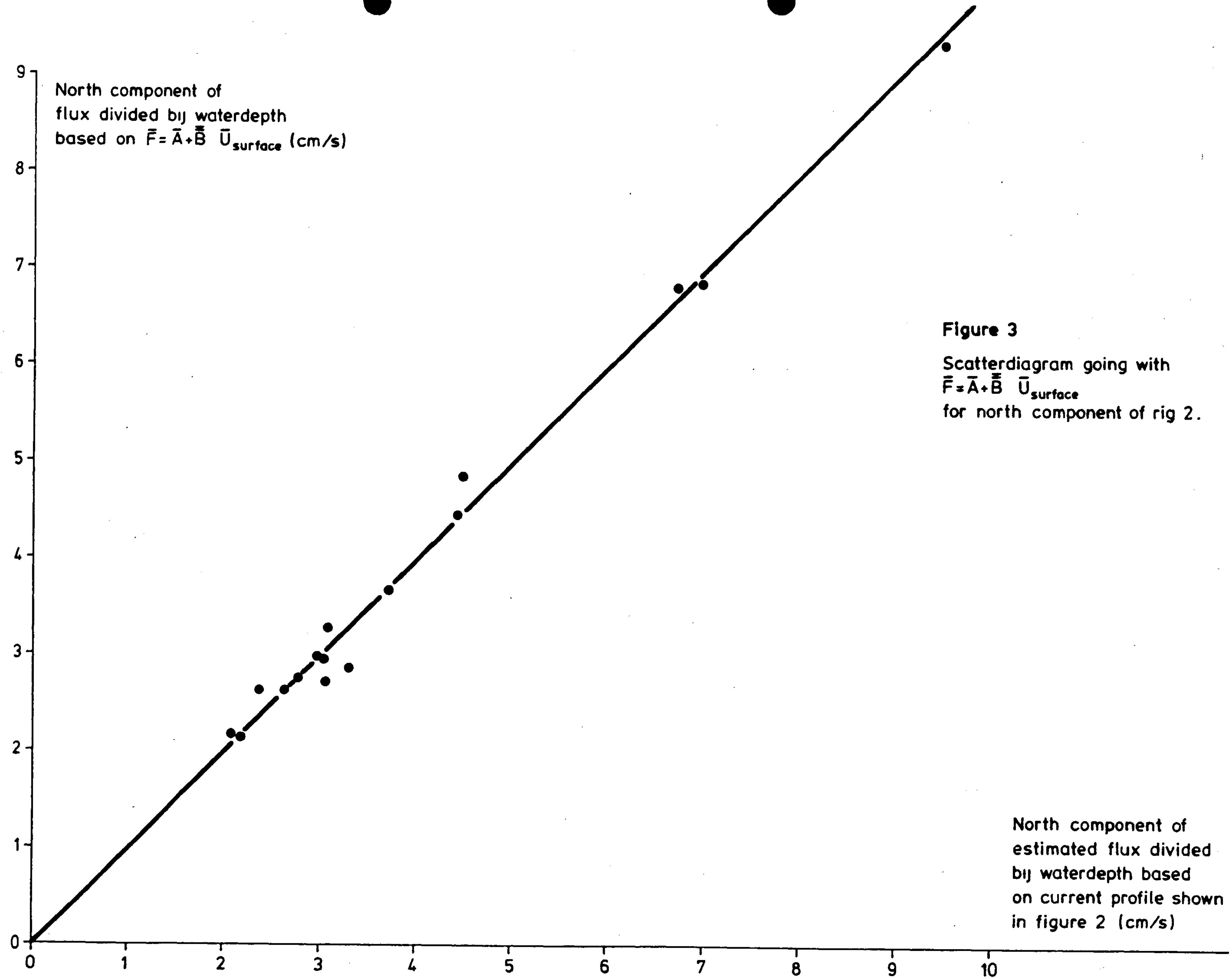

Figure 3

Scatterdiagram going with
$\bar{F} = \bar{A} + \bar{\bar{B}}\ \bar{U}_{surface}$
for north component of rig 2.

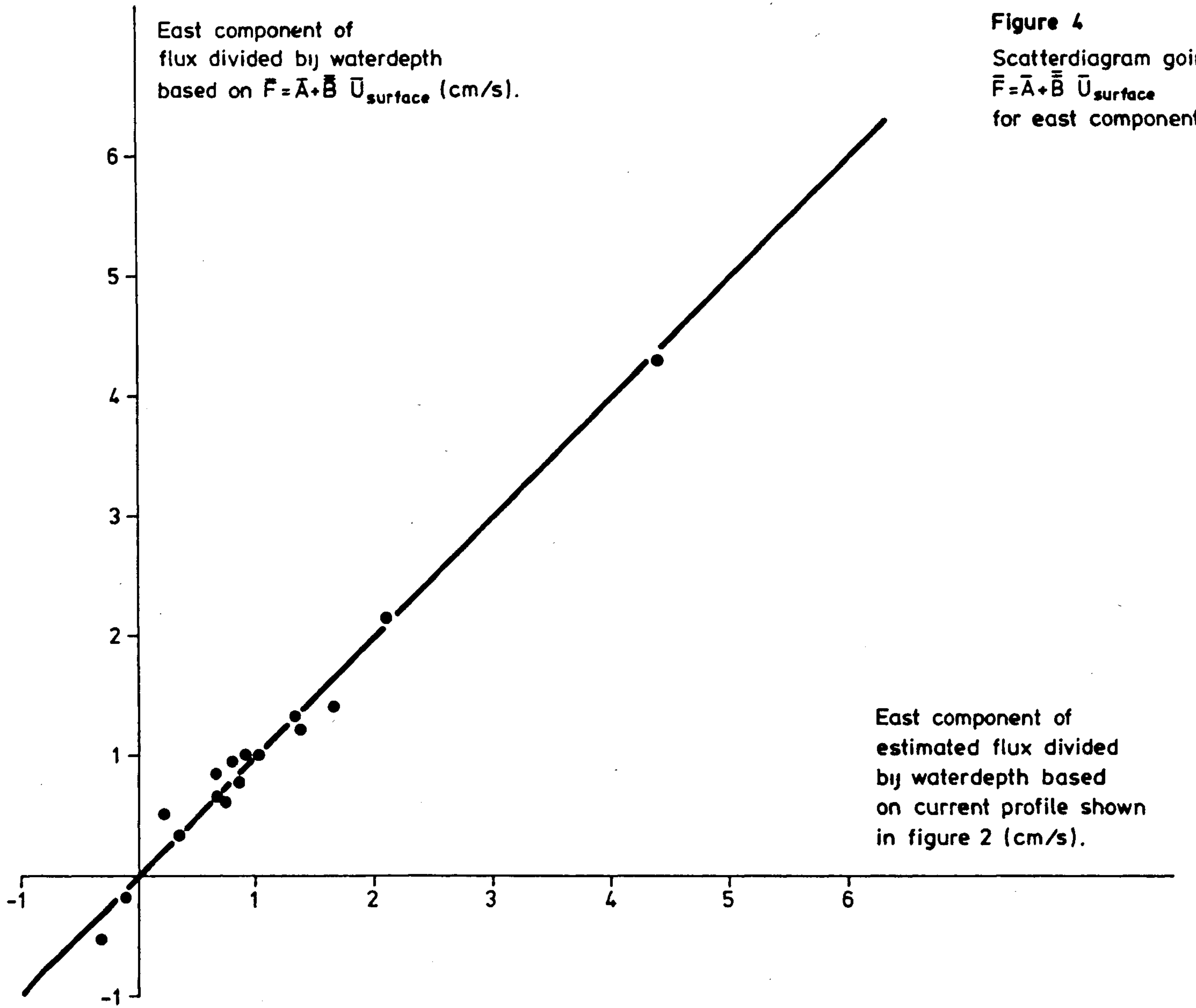

Figure 4

Scatterdiagram going with
$\bar{F} = \bar{A} + \bar{\bar{B}}\ \bar{U}_{surface}$
for east component of rig 2.

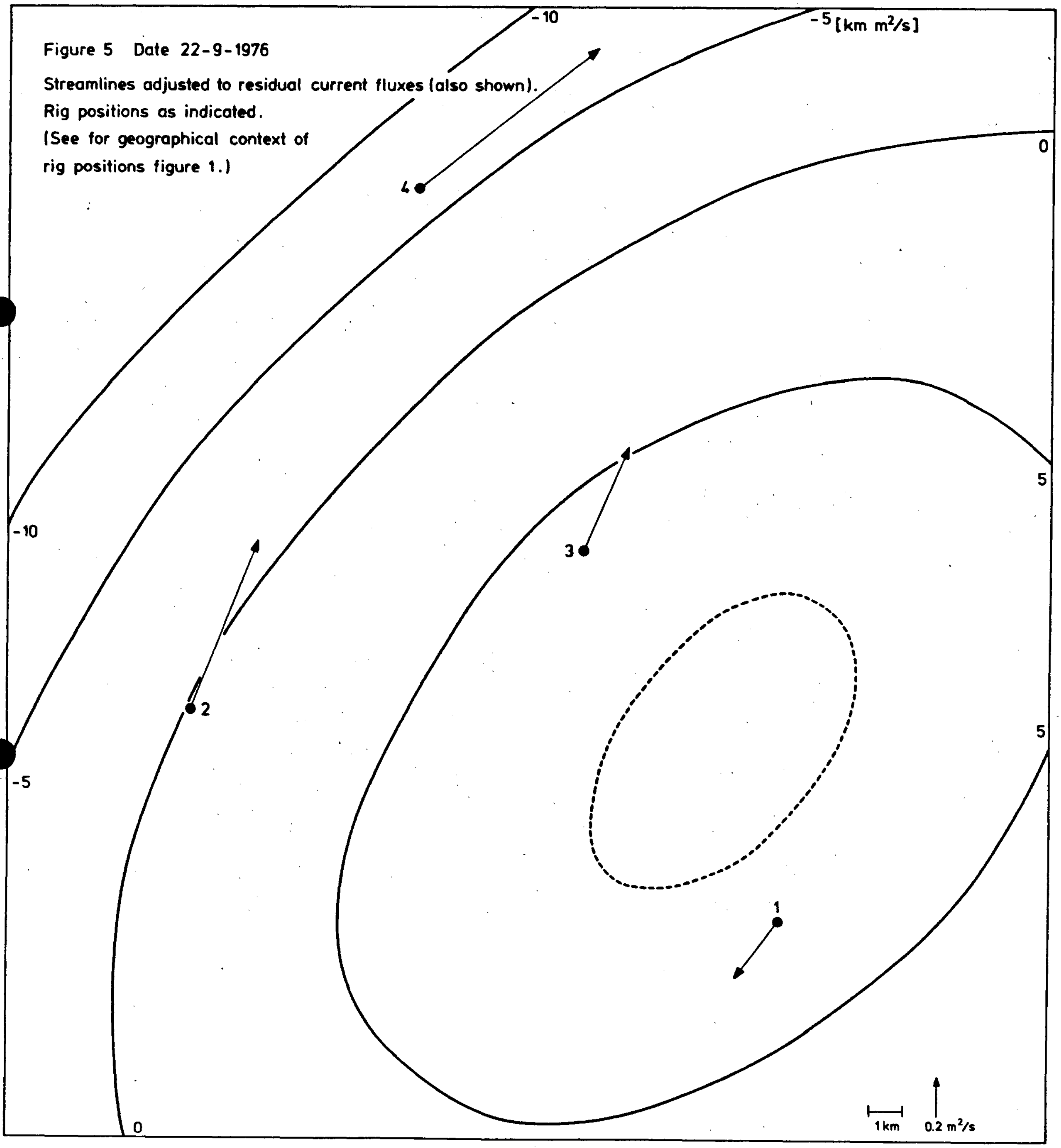

Figure 5 Date 22-9-1976
Streamlines adjusted to residual current fluxes (also shown).
Rig positions as indicated.
(See for geographical context of
rig positions figure 1.)
-10
-5 [km m²/s]
0
5
5
-10
-5
0
1
2
3
4
1 km
0.2 m²/s

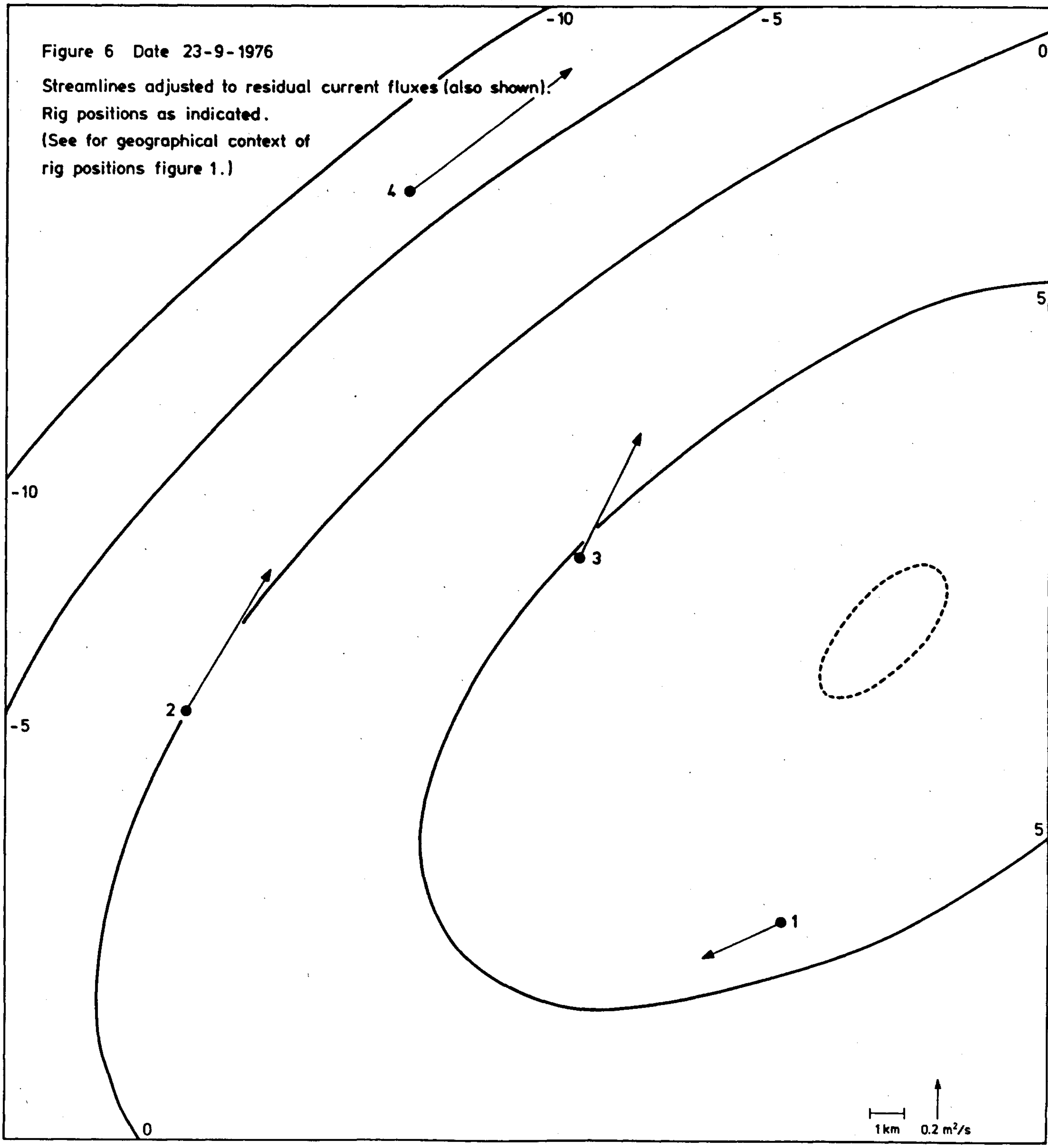

Figure 6 Date 23-9-1976

Streamlines adjusted to residual current fluxes (also shown).
Rig positions as indicated.
(See for geographical context of rig positions figure 1.)

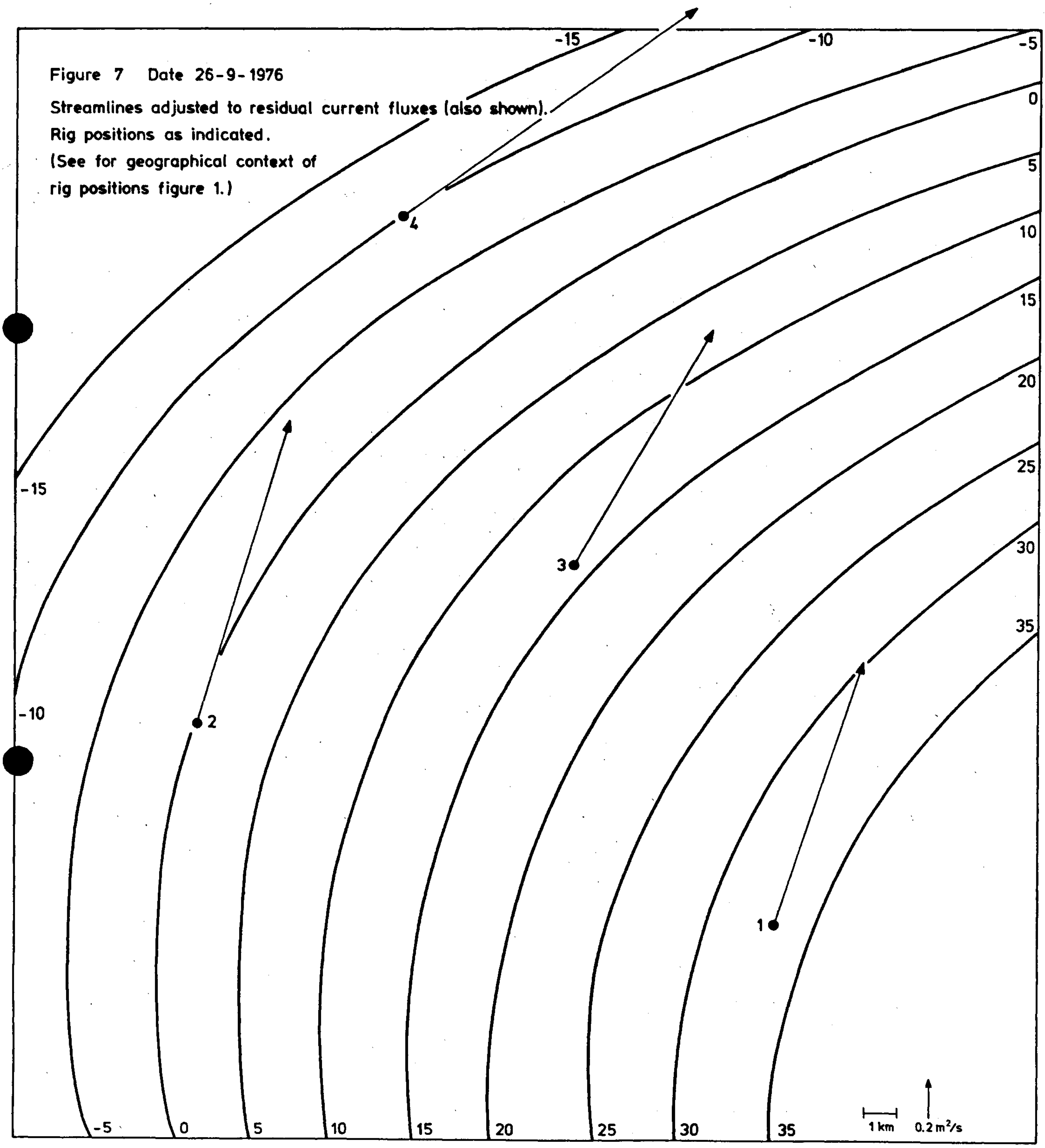

Figure 7 Date 26-9-1976

Streamlines adjusted to residual current fluxes (also shown).
Rig positions as indicated.
(See for geographical context of rig positions figure 1.)

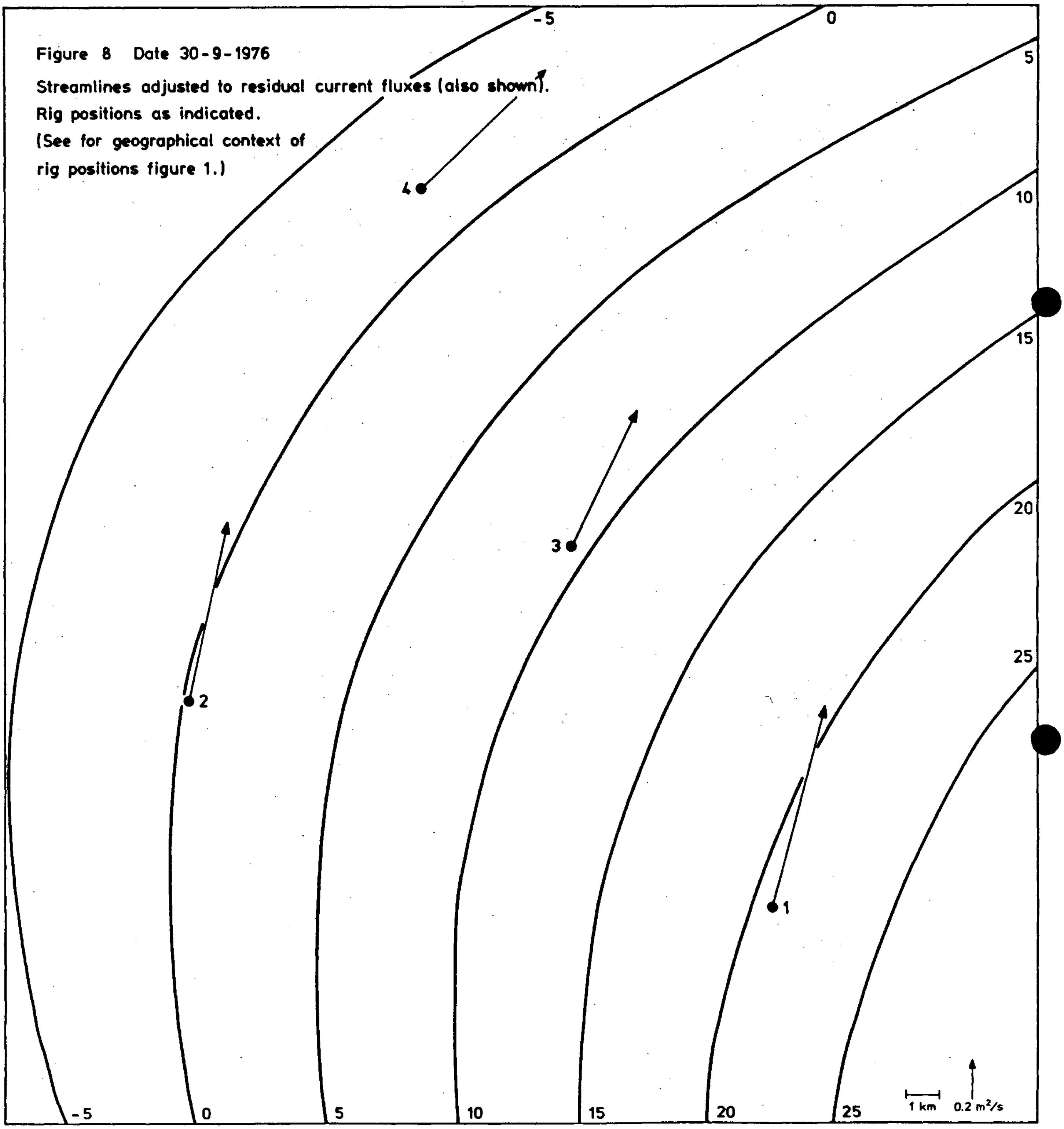

Figure 8 Date 30-9-1976
Streamlines adjusted to residual current fluxes (also shown).
Rig positions as indicated.
(See for geographical context of rig positions figure 1.)
-5
0
5
10
15
20
25
-5
0
5
10
15
20
25
1
2
3
4
1 km
0.2 m²/s

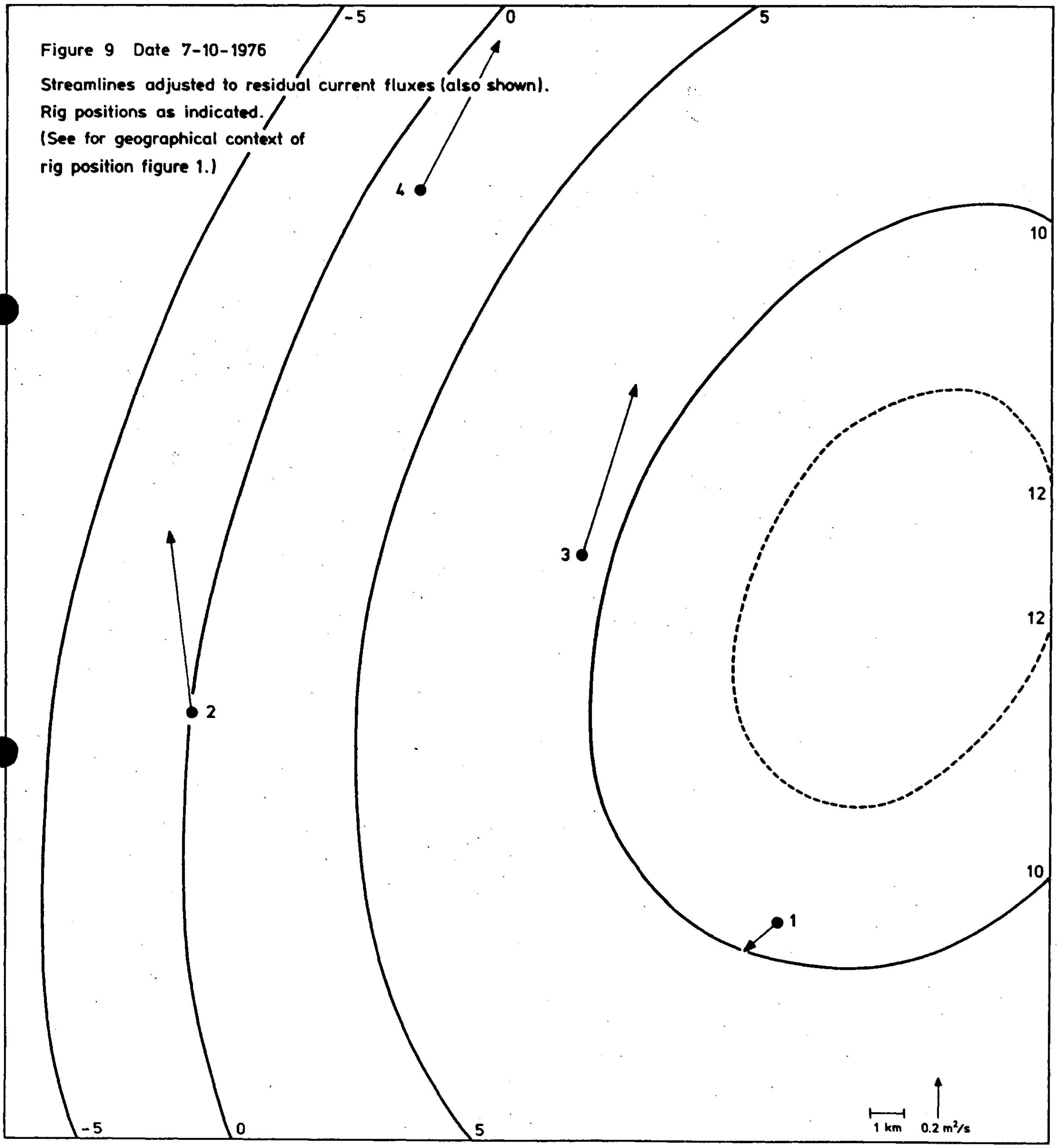

Figure 9 Date 7-10-1976

Streamlines adjusted to residual current fluxes (also shown).
Rig positions as indicated.
(See for geographical context of
rig position figure 1.)

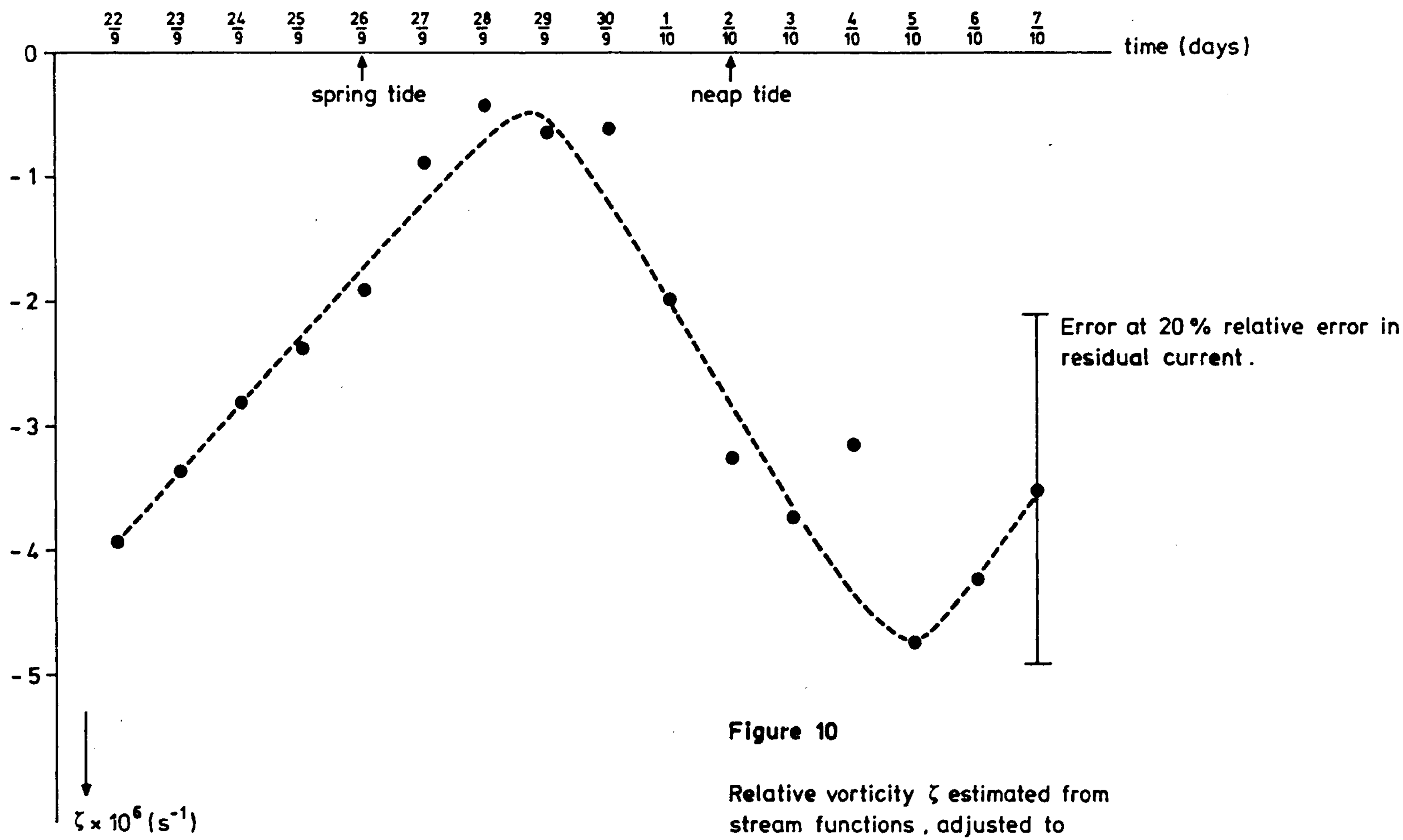

Figure 10

Relative vorticity ζ estimated from stream functions, adjusted to near surface residual currents.

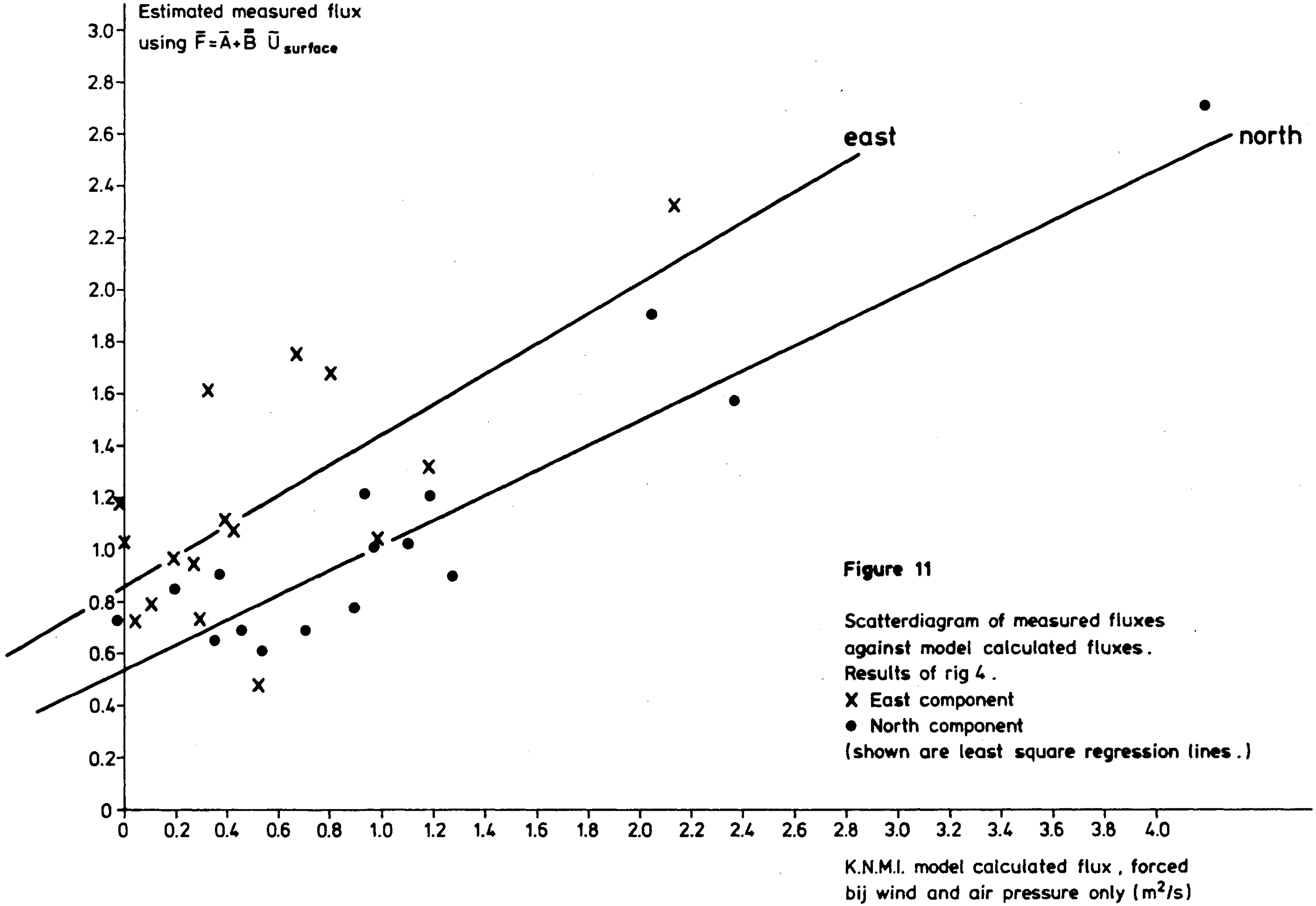

Figure 11

Scatterdiagram of measured fluxes against model calculated fluxes. Results of rig 4.
X East component
● North component
(shown are least square regression lines.)